**UNIT – III FINITE IMPULSE FILTERS**

Design of FIR filters - symmetric and Anti-symmetric FIR filters - design of linear phase FIR filters using Fourier series method - FIR filter design using windows (Rectangular, Hamming and Hanning window), Frequency sampling method. FIR filter structures - linear phase structure, direct form realizations

**Introduction:**

* A digital filter is just a filter that operates on the digital signals.

**Types:**

* FIR filter design
* IIR filter design

**FIR filter:**

The digital filter which designed using finite number of response co-efficient is called as finite impulse response filters.



**Advantages:**

1. FIR filters have exact linear phase.
2. FIR filters are always stable.
3. FIR filters can be realized in both recursive and non-recursive structure.
4. FIR filters with any arbitrary magnitude response can be tackled using FIR sequence.

**Disadvantages:**

1. For the same filter specification the order of the FIR filter design can be as high as 5 to 10 times that of an IIR filter.

2. Large storage requirement needed.

3. Powerful computational facilities required for the implementation.

**Linear Phase (LP) FIR Filters:**

**Derive the condition for Linear Phase (LP) FIR Filters. [Nov/Dec-2009]**

The transfer function of a FIR causal filter is given by



Where *h(n)* is the impulse response of the filter.

The Fourier transform of h(n) is

,

Which is periodic in frequency with period 2.



Where is magnitude response and ( ) is phase response.

We define the phase delay and group delay of a filter as

---------------------------------------->(1)

For FIR filters with linear phase we can define

------------------------------------------------>(2)

Where  is a constant phase delay in samples.

Substitute: equation 2 in 1, we have p = g=, which means that  is independent of frequency. We can write,



Which gives us,

 ------------------------------->(3)

and ---------------------------------->(4)

By taking ratio of equation (3) to equation (4), we obtain

------------------------------>(5)

After simplifying equation (5) we have

------------------------------------------------->(6)

Equation (6) will be zero when

------------------------------------------------------->(7)

And ------------------------------------------------------------------>(8)

Therefore, FIR filters will have constant phase and group delays when the impulse response is symmetrical about 

The impulse response satisfying equation (7) & (8) for odd and even values of N. When N=7 the centre of symmetry of the sequence occurs at third sample and when N=6, the filter delay is samples.

If only constant group delay is required, and not the phase delay we can write

( )= - 

Now we have 

Equation (9) can be expressed as

-------------------------------------->(9)

which gives us

---------------------------->(10)

and ------------------------------>(11)

By taking ratio of equation (11) to (10), we get



From which we obtain

------------------------------------------>(12)

If , Equation (12) becomes,

------------------------------------------------->(13)

The equation 13 will be satisfied when 

And 

Therefore, FIR filters have constant group delay, and not constant phase delay when the impulse response is anti-symmetrical about.

**Example:**

For N=6 ****



0 1 2 2.5 3 4 5

For N=7

For N=7 ****



0 1 2 3 4 n

**Linear Phase FIR Filter:**

An FIR filter has linear phase if its unit sample response satisfies the condition

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**Case (i): Symmetric impulse response for “N is ODD”:**

**Determine the frequency response of FIR filter with symmetric impulse response and the order of the filter is “N is Odd”.**

The frequency response of impulse response can be written as,

-------------------------->(1)



--------->(2)











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**Case (ii) : Symmetric Impulse Response For –“N is EVEN”:**

**Determine the frequency response of FIR filter with symmetric impulse response and the order of the filter N is Even. [Nov/Dec-2013]**

**“N is Even”.**

The frequency response of impulse response can be written as,

---------------------------------------------->(1)



------------------------------>(2)



---------------->(3)









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**Case (iii) : Antisymmetric for “N is ODD”:**

**Determine the frequency response of FIR filter with Antisymmetric impulse response and the order of the filter N is Odd.**

For this type of sequence





The frequency response of impulse response can be written as,















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**Case (iv) : Antisymmetric For –“N is EVEN”:**

**Determine the frequency response of FIR filter with Antisymmetric impulse response and the order of the filter N is Even. [Nov/Dec-2013]**

The frequency response of impulse response can be written as,

















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**Structures of FIR Filters:**

**Explain with neat sketches the Structure of FIR filters. [Nov/Dec-2012]**

The realization of FIR filter is given by

* Transversal structure.
* Linear phase realization
* Polyphase realization.

**Transversal structure:** It contains two forms of realization such as,

* Direct form realization
* Cascade form realization.

**Direct form realization:**

The system function of an FIR filter can be written as



This structure is known as direct form realization. It requires N multipliers, N-1 adders, and N-1 delay elements.









X(z)

h(0)

h(1)

h(2)

h(N-2)

h(N-1)

Y(z)

**Cascade Realization:**

**Problem 1: Determine the direct form Realization of the following system function. (Nov/Dec-14)**

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**Solution:**

Given: The system function is ****

****

Y(z)









X(z)

1

2

6

4

5

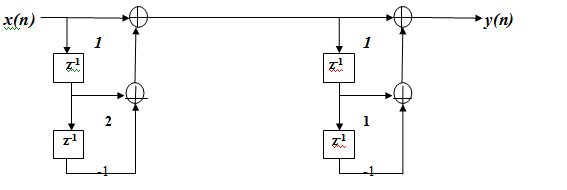
Z-1

**Problem 2: Obtain the cascade realization of system function (May/June-12) (Nov/Dec-10)**

**Solution:**



The equation (1) and equation (2) can be realized in direct form and can be cascaded as shown in figure.



**H.W 1 : Obtain the direct form realization for the following system function.**



**H.W 2: Obtain the cascade form realization for the following system function.**



**Obtain the linear phase realization of the system function. [Nov/Dec-10]**

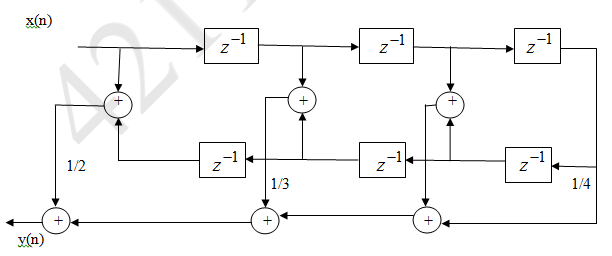
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**Solution:**

By inspection we find system function H(z) is that of a linear phase FIR filter and,

h(n)=h(N-1-n)

Therefore, we can realize the system function as shown in Figure.



**Lattice Structure:**

The lattice structure formulas are,

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**Consider an FIR lattice filter with co-efficients  . Determine the FIR filter for the direct form structure. [Nov/Dec-2013] [Nov/Dec-2015]**

**Solution:**

Given: The FIR lattice filter with co-efficients are 



We know,



*For m=2 and K=1*



*For m=3 and K=1*



*For m=3 and K=2*



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**H.W: Realize the following system in lattice form. [May/June-07]**



**Design of FIR Filter:**

FIR Filter can be designed using three following techniques.

1. Fourier series method
2. Windowing technique
3. Frequency sampling method.

**Filter design using windowing technique:**

**Explain the designing of FIR filters using windows. [April/May-2011]**

The desired frequency response of any digital filter is periodic in frequency and can be expanded in a Fourier series.

------------------------------------------------------>(1)



------------------------------------------------>(2)

**Gibb’s Phenomenon:**

One possible way of finding an FIR filter that approximates would be truncate the infinite Fourier series at . Abrupt truncation of the series will lead to oscillation both pass band and stop band. This phenomenon is known as Gibbs phenomenon.

**Types of window:**

* Rectangular window.
* Hanning window.
* Hamming window.

**Rectangular window:**

The rectangular window sequence is given by,



**Hanning window:**

The hanning window sequence can be obtained by



**Hamming window:**

The hamming window can be obtained by



**Filter coefficient (hd(n) ) for different types of Filters:**

|  |  |
| --- | --- |
| **Type of Filter** | **hd(n)** |
| **LPF** |  |
| **HPF** |  |
| **BPF** |  |
| **BSF** |  |

**Design an ideal low pass filter with a frequency response**

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**Find the values of *h(n)* for M=11 using hanning window. Find H(z). Plot the magnitude and frequency response. (May/June-14)(Nov/Dec-14) (April/May 2011)(April/May-08) (Nov/Dec-09) (Nov/Dec-10)**

**Solution:**

**Given:**

** **

**Step 1: To find filter coefficient.**





 ****

 ****

 ****

**Step 2: To find hanning window:**





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**Step 3: To find filter coefficients using hanning window are**











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**Step 4: The transfer function of the filter is given by**





**Magnitude in d**B is calculated by varying 0 to 10 and tabulated below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 |
|  | 0.812 | 0.8115 | 0.810 | 0.8083 | 0.8054 | 0.8018 |
|  | -1.8 | -1.814 | -1.83 | -1.85 | -1.88 | -1.91 |

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**Design an ideal high pass filter with a frequency response**

**Find the values of *h(n)* for N=11 using hanning window. (May/June-16)(April/May-08)**

**Solution:**

**Given :**

****



**Step 1: To find filter coefficient.**





,  

Step 2: Using Hanning window:















**Step 3: The filter coefficients using hanning window are,**















**Step 4: The transfer function of the filter is given by**











































|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 |
|  | 0.082 | 0.0822 | 0.083 | 0.08433 | 0.08615 | 0.08848 |
|  | -21.72 | -21.70 | -21.61 | -21.480 | -21.29 | -21.11 |

**b) Using Hamming window:**

**The hamming window sequence is given by**











































































|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **0** | **30** | **60** | **90** | **120** | **150** | **180** |
|  | **0.07** | **0.28** | **0.7168** | **0.9668** | **1** | **1.003** | **1.0108** |
|  | **-23.1** | **-11** | **-2.89** | **-0.29** | **0** | **0.028** | **0.093** |

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**H.W: 1. Design a filter with . Using a Hamming & Hanning window with N=7**

**2. Design a filter with . Using a Hamming & Hanning window with N=11.**

**3. Design an FIR filter for the ideal frequency response using hamming window with N=7.**

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**For a FIR linear phase digital filter approximating the ideal frequency response**

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**Determine the coefficients of a 5 tap filter using rectangular window.**

**Solution:**

**Given:**

**** ****

**Step 1: To find filter coefficient.**



For n=0:  



****  ****

 ****

**Step 2: Using Rectangular window:**





**Step 3: To find filter coefficients using rectangular window are**







****

**Step 4: The transfer function of the filter is given by**



























|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **0** | **30** | **60** | **90** | **120** | **150** | **180** |
|  | **1.774** | **1.47** | **0.773** | **0.046** | **-0.407** | **-0.564** | **-0.586** |

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**Design an ideal band pass filter with a frequency response**

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**Find the values of *h(n)* for N=11 using rectangular window.**

**Solution:**

Given:

****

**Step 1: Filter coefficients are,**



**Step 2: Using rectangular window**





**Step 3: Filter coefficients using rectangular window**



**Step 4: The transfer function of the filter is**

**Step 5: The transfer function of the realizable filter is**



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|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **0** | **30** | **60** | **90** | **120** | **150** | **180** |
|  | **-0.1366** | **0.1817** | **0.818** | **1.1366** | **0.818** | **0.1817** | **-0.1366** |
|  | **-17.3** | **-14.8** | **-1.74** | **1.11** | **-1.74** | **-14.8** | **-17.3** |

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**Design an ideal band Reject filter with a frequency response Find the values of *h(n)* for N=11 using rectangular window.**

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**(May/June-14)(Nov/Dec-12)(Nov/Dec-2011)**

**Solution:**

Given:

****

**Step 1: Filter coefficients are,**



**Step 2: Using rectangular window**





**Step 3: Filter coefficients using rectangular window**



**Step 4: The transfer function of the filter is**



**Step 5: The transfer function of the realizable filter is**



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|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **0** | **30** | **60** | **90** | **120** | **150** | **180** |
|  | **0.9428** | **1.08** | **0.526** | **16** | **0.529** | **1.08** | **0.9428** |
|  | **-0.5** | **0.67** | **-5.53** | **-15.9** | **-5.53** | **0.67** | **-0.5** |

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**Design a high pass filter using window, with a cut-off frequency of 1.2 radians/sec and N=9. [Nov/Dec-2016]**

Solution:

Given



The impulse response of a high pass filter with a cut off frequency is





Hamming window for 







The casual filter coefficients are



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**H.W: Design an ideal differentiator with frequency response  using hamming window with N=8. (April/May-15)**

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**Frequency Sampling Method:( April/May-15)**

**Discuss the design procedure of FIR filters using frequency sampling method. [May/June-2013]**

Generally, FIR filter can be specified by giving impulse response coefficients *h(n)* (or) DFT coefficients H(k).

-------------------------------> (1)

----------------------------------> (2)



--------------------------------------> (3)

 --------------------------------------> (4)

**General steps to design FIR filter using frequency sampling method [type-I design]:**

Step 1: Draw the filter graph, as in FIR design using window function.

Step 2: Draw the unit circle and mark the points, if k-0,1…N-1.



Step 3: To find H(k), replace is the given equation.



Step 4: Find h(n)



Step 5: Find H(z):



**Design LPF which has the following specifications, N=7 using frequency sampling Technique. [Nov/Dec-2016][Nov/Dec-15]**

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**Solution:**

Given: 

Step 1: From a unit circle, mark points from





Step 2: 



N=7.



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**Find coefficient of LP FIR with N=15 and it has symmetric unit sample response. It satisfies the following condition.**





**Solution:**

**Step 1:**

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**Step 2:**













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**Determine the coefficients {h (n)} of a linear phase FIR filter of length M =15 has a symmetric unit sample response and a frequency response that satisfies the condition (May/June-13) (April/May-11)(Nov/Dec-09)**

**Solution:**











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**Using frequency sampling method, design BPF with the following specifications. [May/June-2016]**

**Sampling frequency F=8000Hz**

**Cut off frequencies fc1=1000Hz**

**Cut off frequencies fc2=3000Hz Determine the filter coefficients for N=7.**

**Solution:**

 *k=0,1,…6*



The filter coefficients are given by

*  







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**H.W:1. Design a linear phase FIR high pass filter using hamming window, with a cutoff frequency, ωc=0.8π rad/sample and N=7.**

**2. Design a FIR low pass filter with cutoff frequency of 1KHz and sampling frequency of 4 kHz with 11 samples using Fourier series method. Determine the frequency responses and verify the design by sketching the magnitude responses**

**3. Using a rectangular window technique design a low pass filter with pass band gain of unity, cutoff frequency of 1000Hz and working at a sampling frequency of 5kHz.the length of the impulse response should be 7.**

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